Vibration Analysis of Simply Supported Parallelogram Plate with Bi-Dimensional Thickness and Temperature Deviation

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Abstract- In present paper authors represented a simple model to study the effect of non-homogeneity on vibration of parallelogram plate with bi-linear thickness variation. Thermal induced vibration of these plates has been assumed as bi-linear temperature circulation. For non-homogeneity of the plate material density is taken to be linearly varying. The governing differential equation has been solved with the help of variables separation method. The approximated frequency equation is derived by using Rayleigh-Ritz method by two term deflection function. The frequency values for the first two modes of vibration have been calculated for a simply supported parallelogram plate for various values of aspect ratio, thermal gradient, skew angle and taper constants.

Index Terms-Vibration, linear thickness, orthotropic, non-homogeneity, parallelogram plate, thermal gradient.

1. INTRODUCTION

In the engineering, the entire machines and engineering structures experiences vibrations so we cannot move further without taking into consideration the effect of vibration. The requirement to know the effect of temperature on visco-elastic plates of variable thickness has become crucial with the development of technology. Tapered Plates with uniform and non-uniform thickness and temperature are widely used in automobile sector, aeronautical field, power plants and marine structure etc. Various researchers analyzed the vibration of different plates homogeneous or non-homogeneous having variable thickness and taking into account or not the temperature effect.

An extensive review on linear vibration of plates has been given by Leissa [1] in his monograph and a series of review articles [2] .Tomar and Gupta [3] considered the effect of taper gradient in two dimensions on elastic plates, but not on visco-elastic plates. Tomar and Gupta [4] studied temperature effect on frequency of an rectangular orthotropic plate with variable thickness in one direction. Gupta and Khanna [5] analyzed vibration of a visco-elastic rectangular plate under the effect of linearly varying thickness in both directions. Gupta ,Kumar and Gupta [6] studied the effect of parabolic thickness variations on vibration of visco-elastic orthotropic parallelogram plate. Bhatnagar and Gupta [7] analyzed Vibration analysis of visco-elastic circular plate subjected to thermal gradient. Khanna and Sharma [8] studied the vibration of visco-elastic square plate with variable thickness and thermal gradient. Gupta and Kumar [9] studied thermal effect on vibration of parallelogram plate of bi-direction linearly varying thickness. Singh and Saxena, [10] studied transverse vibration of rectangular plate with bi-dimensional thickness variation. Khanna [11] analyzed a computational calculation of vibrations of square plate by variable thickness with thermal effect in both direction.

Sobotka [12] represented free vibration of visco-elastic orthotropic rectangular plates. Sharma and Sharma [13] presented the mathematical study on vibration of visco-elastic parallelogram plate. Khanna, Arora and Kaur [14] considered the vibrations of visco-elastic non-homogeneous plate with variable thickness and density.

In present paper, the authors have analyzed the bi-linear temperature deviation effect on the vibrations of non-homogeneous parallelogram plates with inconsistent linear thickness in two dimensions. Also, it is supposed that the plate is simply supported (SSSS). Due to temperature deviation, we suppose that non homogeneity occurs in modulus of elasticity. Frequency for the first and second mode of vibration is obtained for various numerical values of tapering constant, non-homogeneity, thermal gradient and aspect ratio. Results are presented in form of graphs and tables.

2. ANALYSIS OF THE MODEL AND SOLUTION

2.1 Material

The parallelogram plate R with skew angle θ and sides a,b be shown in figure 1. Since it is a special case of rectangular plate, we take $\theta = 0^o$. The plate is taken to be orthotropic and non-uniform. Here

$$\xi = x - y\tan\theta,\ \eta = y\sec\theta \tag{1}$$

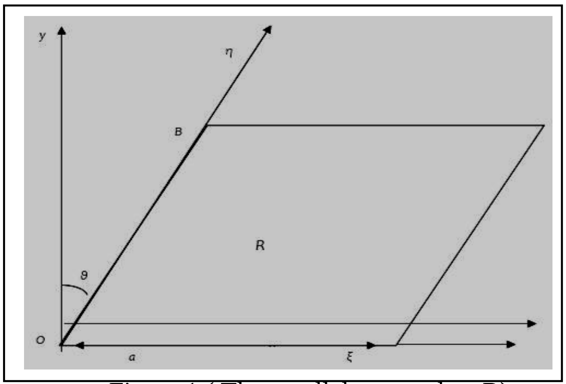

Figure 1.(The parallelogram plate R)

Displacement $W(\xi,\eta,t)$ for free vibration of the parallelogram plate is given by

$$W(\xi,\eta,t) = W(\xi,\eta)T(t) \tag{2}$$

Here $W(\xi,\eta)$ is the maximum displacement at time t and T(t) is the time function.
The plate considered here is subjected to linear temperature distribution along ξ- and η- directions, then

$$\tau = \tau_0\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right) \tag{3}$$

where 'a' represents length, 'b' represents breadth and τ_0 is temperature at origin of the plate.
For orthotropic material, the temperature dependent modulus of elasticity is taken as

$$\begin{aligned} E_\xi(\tau) &= E_1\,(1 - \gamma\,\tau), \\ E_\eta(\tau) &= E_2\,(1 - \gamma\,\tau), \\ G_{\xi\eta}(\tau) &= G_0\,(1 - \gamma\,\tau) \end{aligned} \tag{4}$$

where E_ξ and E_η are Young's moduli in ξ- and η- directions respectively, $G_{\xi\eta}$ is shear modulus and γ is taken as slope variation of moduli with temperature. Using eqn. (3) in eqn. (4) one has

$$\left.\begin{aligned} E_\xi(\tau) &= E_1\left[1 - \alpha\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)\right] \\ E_\eta(\tau) &= E_2\left[1 - \alpha\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)\right] \\ G_{\xi\eta}(\tau) &= G_0\left[1 - \alpha\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)\right] \end{aligned}\right\} \tag{5}$$

Where $\alpha = \gamma\,\tau_0, (0 \leq \alpha < 1)$ is a thermal gradient.
The plate's thickness variation for the present study is to be assumed linearly in ξ- and η- directions which is represented by

$$h = h_0\left[\left(1 + \beta_1\frac{\xi}{a}\right)\left(1 + \beta_2\frac{\eta}{b}\right)\right] \tag{6}$$

Here β_1 and β_2 are known as tapering constants in ξ- and η- directions respectively and
$h = h_0$ at $\xi, \eta = 0$.
The flexural rigidities (D_ξ, D_η) and torsional rigidity ($D_{\xi\eta}$) of the plate are taken as

$$\left.\begin{aligned} &D_\xi = \frac{E_\xi h^3}{12(1 - \nu_\xi\nu_\eta)},\ D_\eta = \frac{E_\eta h^3}{12(1 - \nu_\xi\nu_\eta)} \\ &D_{\xi\eta} = \frac{G_{\xi\eta}h^3}{12},\ D_1 = \nu_\xi D_\eta = \nu_\eta D_\xi \\ &H = D_1 + 2D_{\xi\eta} \end{aligned}\right\} \tag{7}$$

Where ν_ξ, ν_η are Poisson's ratio.
Using eqns. (5) and (6) in eqn. (7), we have

$$\begin{aligned} D_\xi &= \frac{E_1 h_0^3}{12(1-\nu_\xi\nu_\eta)}\left[\left\{1 - \alpha\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)\right\}\left\{\left(1 + \beta_1\frac{\xi}{a}\right)\left(1 + \beta_2\frac{\eta}{b}\right)\right\}^3\right] \\ D_\eta &= \frac{E_2 h_0^3}{12(1-\nu_\xi\nu_\eta)}\left[\left\{1 - \alpha\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)\right\}\left\{\left(1 + \beta_1\frac{\xi}{a}\right)\left(1 + \beta_2\frac{\eta}{b}\right)\right\}^3\right] \\ D_{\xi\eta} &= \frac{G_0 h_0^3}{12}\left[\left\{1 - \alpha\left(1 - \frac{\xi}{a}\right)\left(1 - \frac{\eta}{b}\right)\right\}\left\{\left(1 + \beta_1\frac{\xi}{a}\right)\left(1 + \beta_2\frac{\eta}{b}\right)\right\}^3\right] \end{aligned} \tag{8}$$

For non-homogeneous material, linear variation taken in density is

$$\rho = \rho_0\left(1 - c_1\frac{\xi}{a}\right) \tag{9}$$

Where c_1 $(0 \leq c_1 < 1)$ is non-homogeneity constant.

2.2 Frequency equation and boundary condition

Boundary conditions for a non-homogeneous orthotropic (SSSS) parallelogram plate are taken as

$$\left.\begin{array}{l} W = W_{,\xi\xi} = 0 \text{ at } \xi = 0, a \\ W = W_{,\eta\eta} = 0 \text{ at } \eta = 0, b \end{array}\right\} \quad (10)$$

Two-term deflection function, satisfying the boundary conditions, can be taken as

$$W = \left[\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)\left(1-\frac{\xi}{a}\right)\left(1-\frac{\eta}{b}\right)\right] \left[A_1 + A_2\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)\left(1-\frac{\xi}{a}\right)\left(1-\frac{\eta}{b}\right)\right] \quad (11)$$

where A_1, A_2 are constants to satisfy boundary conditions.

Now, unit less variables having no dimension are using for our convince as

$$\left.\begin{array}{l} X = \frac{\xi}{a}, Y = \frac{\eta}{b}, \hat{W} = \frac{W}{a}, H = \frac{h}{a} \\ E_1^* = \frac{E_1}{1-v_\xi v_\eta}, \quad E_2^* = \frac{E_2}{1-v_\xi v_\eta}, \end{array}\right\} \quad (12)$$

$$E^* = v_\xi E_2^* = v_\eta E_1^* \quad (13)$$

Components of E_1^*, E_2^* , E^* and G_0 are E_1^*, $E_2^* \sec\theta$, $E^* \sec\theta$ and $G_0 \sec\theta$ respectively in ξ- and η-directions.

The expressions for strain energy (V_E) and kinetic energy (T_E) are taken as

$$V_E = \frac{1}{2}\int_0^a\int_0^b \Big[D_\xi (W_{,\xi\xi})^2 + D_\eta (W_{,\xi\xi}\tan^2\theta - 2W_{,\xi\eta}\sec\theta\tan\theta + W_{,\eta\eta}\sec^2\theta)^2 + 2D_1 W_{,\xi\xi}(W_{,\xi\xi}\tan^2\theta + 2W_{,\xi\eta}\sec\theta\tan\theta + W_{,\eta\eta}\sec^2\theta) + 4D_{\xi\eta}(-W_{,\xi\xi}\tan\theta + W_{,\xi\eta}\sec\theta)^2\Big]\cos\theta \, d\eta d\xi \quad (14)$$

and

$$T_E = \frac{1}{2} p^2 \int_0^a\int_0^b (\rho h W^2 \cos\theta) d\eta d\xi \quad (15)$$

2.2 Solution by Rayleigh-Ritz Method

Rayleigh – Ritz method is used to find an appropriate vibrational frequency. This method works on the phenomena that the maximum strain energy (V_E) must equal to maximum kinetic energy (T_E). An equation in the following form is obtained as

$$\delta(V_E - T_E) = 0 \quad (16)$$

Using eqns. (8), (12), (13) in eqn. (14) and (15), then substituting the values of V_E and T_E in Eqn. (16), we obtained

$$\delta(V_E^* - \lambda^2 T_E^*) = 0 \quad (17)$$

Here,

$$V_E^* = \int_0^1\int_0^{b/a} [\{1-\alpha(1-X)(1-Y)\}\{(1+\beta_1 X)(1+\beta_2 Y)\}^3] \Big[\Big\{\cos^4\theta + \frac{E_2^*}{E_1^*}\sin^4\theta + 2\frac{E^*}{E_1^*}\sin^2\theta\cos^2\theta + 4\frac{G_0}{E_1^*}\sin^2\theta\cos^2\theta\Big\}\hat{W}^2_{,XX} + \frac{E_2^*}{E_1^*}\hat{W}^2_{,YY} + 4\Big\{\frac{E_2^*}{E_1^*}\sin^2\theta + \frac{G_0}{E_1^*}\cos^2\theta\Big\}\hat{W}^2_{,XY} + 2\Big\{\frac{E_2^*}{E_1^*}\sin^2\theta + \frac{E^*}{E_1^*}\cos^2\theta\Big\}\hat{W}_{,XX}\hat{W}_{,YY} - 4\Big\{\frac{E_2^*}{E_1^*}\sin^3\theta + 2\frac{E^*}{E_1^*}\sin\theta\cos^2\theta + 2\frac{G_0}{E_1^*}\sin\theta\cos^2\theta\Big\}\hat{W}_{,XX}\hat{W}_{,XY} - 4\Big\{\frac{E_2^*}{E_1^*}\sin\theta\Big\}\hat{W}_{,YY}\hat{W}_{,XY}\Big] dYdX \quad (18)$$

$$T_E^* = \int_0^1\int_0^{b/a} [(1-c_1X)\{(1+\beta_1X)(1+\beta_2Y)\}] \hat{W}^2 \, dYdX \quad (19)$$

and frequency $\lambda^2 = \frac{12p^2\rho_0 a^2\cos^5\theta}{E_1^* h_0^2}$

Now, the value of A_1 & A_2 is to be determined from (17) as

$$\frac{\partial(V_E^* - \lambda^2 T_E^*)}{\partial A_s} = 0, \text{ for } s = 1, 2 \quad (20)$$

On solving equation (20), we have

$$m_{s1}A_1 + m_{s2}A_2 = 0, \text{ for } s = 1, 2 \quad (21)$$

Here m_{s1}, m_{s2} (s = 1,2) comprises parametric constant and the frequency parameter.

The determinant of the co-efficient of equation (21) must be zero, for non-trivial solution,

we get the equation of frequency as follows

$$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = 0 \quad (22)$$

With the help of equation (22), we get quadratic equation in λ^2. We can obtain two roots of λ^2 from this equation. These roots give the first (λ_1) and second (λ_2) modes of vibration of frequency for various parameters of tapering constants, thermal gradient and aspect ratio for a simply supported plate.

3. RESULT AND DISCUSSION

The frequency (λ) for 1^{st} and 2^{nd} mode of vibration of an orthotropic (simply supported) parallelogram plate has been calculated for various values of thermal constant (α), tapering constant (β_1 and β_2), aspect ratio (a/b) and non-homogeneity constant (c_1). All the results are obtained by using MATLAB / MAPPLE software. Following parameters are used for these calculations [6]:

$$\frac{E_2^*}{E_1^*} = 0.01,\ \frac{E^*}{E_1^*} = 0.3,\ \frac{G_0}{E_1^*} = 0.0333,\ \frac{E_1^*}{\rho} = 3.0 \times 10^5,\ h_0 = 0.01m \text{ and } \rho_0 = 0.345$$

The results are given in tables [1-5].

Table-1 represents thermal gradient (α) versus frequency (λ) with fixed value of aspect ratio (a/b = 1) and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = c_1 = 0$, 0.4, 0.8). It is evident from Table-1 that as value of thermal gradient (α) increases from 0 to 0.8 corresponding frequency value (λ) for 1^{st} and 2^{nd} mode of vibration decreases.

Table-2 represents taper constant (β_1) versus Frequency (λ) with fixed value of aspect ratio (a/b = 1) and different values of thermal gradient, tapering constant and non-homogeneity ($\alpha = \beta_2 = c_1 = 0$, 0.4, 0.8). From Table-2 it is clear that as value of tapering constant (β_1) varies from 0 to 0.8 corresponding frequency value (λ) also increases for 1^{st} and 2^{nd} mode of vibration.

Table-3 represents taper constant (β_2) versus Frequency (λ) with fixed value of aspect ratio (a/b = 1) and three different values of thermal gradient, tapering constants and non-homogeneity ($\alpha = \beta_1 = c_1 = 0$, 0.4, 0.8). From Table-3 it is clear that as the value of tapering constant (β_2) varies from 0 to 0.8 Corresponding value of frequency (λ) also increases for 1^{st} and 2^{nd} mode of vibration.

Table-4 represents non-homogeneity constant (c_1) versus frequency (λ) with fixed value of aspect ratio (a/b = 1) and different values of tapering constants and thermal constant ($\beta_1 = \beta_2 = \alpha = 0$, 0.4, 0.8). It is evident from Table-4 that as value of non-homogeneity constant (c_1) varies from 0 to 0.8 corresponding value of frequency (λ) also increases for 1^{st} and 2^{nd} mode of vibration.

Table-5 represents aspect ratio (a/b) versus frequency (λ) with various values of tapering constants, thermal constant and non-homogeneity ($\beta_1 = \beta_2 = \alpha = c_1 = 0$, 0.4, 0.8). It is evident from Table-5 that as the value of aspect ratio increases from 0 to 1 corresponding value of frequency (λ) for 1^{st} and 2^{nd} mode of vibration decreases.

Table-1. Thermal Gradient (α) vs Frequency (λ)

α	$\beta_1 = \beta_2 = C_1=0, \theta = 30^0$		$\beta_1 = \beta_2 = C_1=0.4, \theta = 45^0$		$\beta_1 = \beta_2 = C_1=0.8, \theta = 60^0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	6.61	36.04	8.68	50.90	9.99	68.83
0.2	6.45	34.93	8.52	50.58	9.84	68.64
0.4	6.27	33.78	8.36	50.28	9.69	68.43
0.6	6.11	32.61	8.19	49.98	9.52	68.23
0.8	5.93	31.36	8.01	49.67	9.33	68.04

Table-2. Taper Constant (β_1) vs Frequency(λ)

β_1	$\alpha = \beta_2 = C_1=0, \theta = 30^0$		$\alpha = \beta_2 = C_1=0.4, \theta = 45^0$		$\alpha = \beta_2 = C_1=0.8, \theta = 60^0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	6.61	36.04	6.73	36.10	6.97	38.63
0.2	7.32	40.45	7.52	42.64	7.83	46.81
0.4	8.09	46.14	8.37	50.29	8.54	55.73
0.6	8.89	52.71	9.22	58.66	9.47	64.12
0.8	9.67	59.85	10.05	67.51	10.63	74.83

Table-3. Taper Constant (β_2) vs Frequency (λ)

β_2	$\alpha = \beta_1 = C_1=0, \theta = 30^0$		$\alpha = \beta_1 = C_1=0.4, \theta = 45^0$		$\alpha = \beta_1 = C_1=0.8, \theta = 60^0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	6.61	36.04	6.97	39.61	6.67	41.97
0.2	7.28	39.78	7.66	44.84	7.32	48.49
0.4	7.96	43.75	8.37	50.29	7.98	55.16
0.6	8.65	47.90	9.08	55.90	8.66	61.95
0.8	9.36	52.16	9.81	61.61	9.34	68.83

Table-4. Non-homogeneity constant (c_1) vs Frequency (λ)

C_1	$\alpha = \beta_1 = \beta_2=0, \theta = 30^0$		$\alpha = \beta_1 = \beta_2=0.4, \theta = 45^0$		$\alpha = \beta_1 = \beta_2=0.8, \theta = 60^0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	6.61	36.04	7.47	44.70	12.44	91.46
0.2	6.96	37.99	7.88	47.25	13.13	96.86
0.4	7.39	40.30	8.37	50.29	13.95	103.34
0.6	7.90	43.08	8.96	54.01	14.95	111.32
0.8	8.53	46.53	9.69	58.68	16.21	121.51

Table-5. Aspect ratio (a/b) vs Frequency (λ)

a/b	$\alpha = \beta_1=\beta_2= C_1=0, \theta = 30^0$		$\alpha =\beta_1=\beta_2=C_1=0.4, \theta=45^0$		$\alpha =\beta_1=\beta_2=C_1=0.8, \theta=60^0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.2	128.43	1018.60	153.14	1352.62	275.53	3057.90
0.4	34.04	250.88	41.21	335.62	75.11	766.60
0.6	16.32	108.74	20.07	146.88	37.12	340.44
0.8	9.87	59.02	12.31	80.79	23.16	190.83
1.0	6.61	36.04	8.36	50.29	16.21	121.51

4. COMPARISON & CONCLUSION

Authors compared the frequency values of present paper (non-homogeneous and SSSS plate) with [13] (homogeneous and CCCC plate) at $c_1 = 0$ (non-homogeneity constant). Authors observed that the frequency values are less in present paper than [13] for same parameters. Comparison tables (Table-6 and Table-6) are given below:

Table-6. Skew angle (θ) vs Frequency (λ)

θ	$a/b = 1, \beta_1 = \beta_2= 0, C_1 = 0, \alpha = 0$		$a/b = 1, \beta_1= \beta_2= 0.4, C_1 = 0, \alpha = 0.4$	
	λ_1	λ_2	λ_1	λ_2
0^o	7.83 (34.01)	43.53 (139.01)	11.03 (40.01)	66.73 (160.99)
30^o	6.60 (48.99)	36.04 (192.93)	9.33 (56.77)	56.77 (222.93)
45^o	5.24 (78.11)	28.04 (301.01)	7.70 (90.03)	44.70 (346.02)

Table-7. Thermal Gradient (α) vs Frequency (λ)

α	a/b = 1, $\beta_1 = \beta_2$ = 0.4, C_1 = 0, θ = 0°		a/b = 1, $\beta_1 = \beta_2$ = 0.4, C_1 = 0, θ = 60°	
	λ_1	λ_2	λ_1	λ_2
0.0	11.52 (40.58)	67.45 (160.98)	5.39 (213.89)	31.02 (804.12)
0.2	11.28 (38.68)	67.28 (154.26)	5.31 (204.62)	30.83 (769.99)
0.4	11.03 (36.92)	66.73 (146.86)	5.23 (194.92)	30.67 (732.62)
0.6	10.77 (34.89)	66.36 (139.87)	5.14 (184.64)	30.50 (692.95)
0.8	10.55 (32.78)	66.01 (131.75)	5.04 (173.86)	30.32 (652.88)

Here, values of [13] are given in brackets. In Table-6 it is evident that as skew angle increases frequency for both modes increases for [13] but decreases for present paper. The present paper describes the behavior of frequencies for 1st and 2nd mode of vibration corresponding to the thermal gradient, tapering constants, aspect ratio and non-homogeneity of the material. The objective of the study is to deliver certain substantial data for frequency modes. The frequency can be optimize by taking suitable variation in parameters. Therefore engineers are advised to analyze the results of present problem and develop the plate's structure in such a way to facilitate the basic requirements.

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